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DILATON-DRIVEN INFLATION IN STRING COSMOLOGY

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ABSTRACT

I present an outline for cosmological evolution in the framework of string theory with emphasis on a phase of dilaton-driven kinetic inflation. It is shown that a typical background of stochastic gravitational radiation is generated, with strength that may allow its detection in future gravity wave experiments.

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INTRODUCTION

I present an outline of cosmological evolution in the framework of string theory. The main emphasis is on a phase of dilaton-driven kinetic inflation and its possible observable consequences, in particular, a background of stochastic gravitational radiation. The results concerning the produced spectrum of gravitational radiation were obtained in [1, 2]. More details on various aspects of the suggested outline and additional references may be found in [3-11].

POTENTIAL-DRIVEN INFLATION

Inflationary evolution of the universe requires a source of energy to drive the expansion. The conventional expectation is that the energy source is dominated by potential energy of scalar fields, called inflatons [12]. The inflatons are expected to possess non-vanishing potential energy during some phase in their evolution in which inflationary expansion takes place. Eventually, the inflatons settle down to the true minimum of their potential where the potential energy vanishes, thus depriving the universe of the necessary source to drive its accelerated expansion. The inflationary phase ends and the universe continues to expand sub-luminally until today. If one tries to implement similar ideas in the framework of string theory, an apparent problem is immediately encountered [13, 14]. String theory does indeed contain many scalar fields, called moduli, which seem particularly suitable for the job of inflatons [15, 16]. Among the moduli the dilaton ϕ is an important and universal field whose expectation value determines the string coupling parameter $g_s^2 \sim \langle \exp(\phi) \rangle$. It couples to all other fields with gravitational strength. If some scalar field, for example, one of the moduli fields acquires a non-vanishing potential so does the dilaton. The type of generated dilaton potential depends on the details of the model. Two types are distinguished, perturbative $V(\phi) \sim \exp(-\alpha\phi/M_{Pl})$, and non perturbative $V(\phi) \sim \exp(-\exp(-\beta\phi/M_{Pl}))$, with particular numerical parameters α, β . The equations of motion for the resulting string dilaton-gravity, assuming isotropic and homogeneous universe

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t)dx_id x^i \\ \phi &= \phi(t), \end{aligned} \tag{1}$$

are the following

$$\begin{aligned} H^2 &= \frac{8\pi}{3M_{Pl}^2} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) \\ \ddot{\phi} + 3H\dot{\phi} &= -\frac{dV}{d\phi} \end{aligned} \tag{2}$$

The Hubble parameter, H , is related to the scale factor, a in the usual way, $H \equiv \frac{\dot{a}}{a}$ and V is the potential. Consider, for example, the (unrealistic) case of exponential potential $V = V_0 \exp(-\alpha\phi/M_{Pl})$ for which can solve eqs. (2) explicitly

$$a(t) = a_0 t^{16\pi/\alpha^2}. \tag{3}$$

If the potential is steeper than the critical steepness $\alpha = 4\sqrt{\pi}$, the dilaton kinetic energy becomes dominant over potential energy and the expansion is subluminal. The generic situation in string theory is that the potentials in several models are steeper than critical and therefore potential-driven inflation requires special situations and is generally speaking hard to obtain. Recently, some progress has been made towards characterizing requirements from models in which potential-driven inflation could be supported [16, 17].

DILATON-DRIVEN KINETIC INFLATION

The outline for cosmological evolution that I present here relies heavily on the fact that the kinetic energy of the dilaton tends to dominate the energy density. Instead of trying to fight this tendency, one accepts it and turns this feature into a virtue, using it to drive kinetic energy dominated inflationary evolution. Kinetic inflation was also discussed in [18]. The evolution starts when the dilaton is deep in the weak-coupling region ($\phi \ll -1$) and Hubble parameter, H , is small. The evolution in this epoch is shown below to be accelerated expansion dominated by the dilaton kinetic energy and determined by the vacuum solution of the string dilaton-gravity equations of motion [3]. To describe the first phase in more detail, look for solutions of the effective string equations of motion in which the metric is of the isotropic, FRW type with vanishing spatial curvature and the dilaton depends only on time. One

finds three independent first order equations for the dilaton and H

$$\dot{H} = \pm H \sqrt{3H^2 + U + e^\phi \rho} - \frac{1}{2}U' + \frac{1}{2}e^\phi p \quad (4a)$$

$$\dot{\phi} = 3H \pm \sqrt{3H^2 + U + e^\phi \rho} \quad (4b)$$

$$\dot{\rho} + 3H(\rho + p) = 0 \quad (4c)$$

where $U = e^\phi V$. Some sources in the form of an ideal fluid were included [3] as well. The (\pm) signifies that either $(+)$ or $(-)$ is chosen for both equations simultaneously. The solutions of equations (4a-4c) belong to two branches, according to which sign is chosen. In the absence of any potential or sources the $(+)$ branch solution for $\{H, \phi\}$ is given by

$$\begin{aligned} H^{(+)} &= \pm \frac{1}{\sqrt{3}} \frac{1}{t - t_0} \\ \phi^{(+)} &= \phi_0 + (\pm\sqrt{3} - 1) \ln(t_0 - t) \quad , \quad t < t_0 \end{aligned} \quad (5)$$

This solution describes either accelerated contraction and evolution towards weak coupling or accelerated inflationary expansion and evolution from a cold, flat and weakly coupled universe towards a hot, curved and strongly coupled one. I assume that the initial conditions are such that the latter is chosen. In general, the effects of a potential and sources on this branch are quite mild. After a period of time, of length determined by the initial conditions, a “Branch Change” event from the dilaton-driven accelerated expansion era into what will eventually become a phase of decelerated expansion has to occur. It occurs either when curvatures and kinetic energies reach the string curvature or when quantum effects become strong enough. The correct dynamical description of this phase should, therefore, be stringy in nature. If the value of the dilaton is small throughout this stage of evolution, dynamics can be described by classical string theory in terms of a two-dimensional conformal field theory. This stage is not yet well understood. At the moment, the only existing examples are not quite realistic [19, 20]. More ideas about this stage may be found in [21, 22]. The value which the dilaton takes at the end of this epoch ϕ_{end} is an important parameter. After the “Branch Change” event, the universe cools down and may be described accurately, again, by means of string dilaton-gravity effective

theory. Now, however, radiation and matter are important factors. The dilaton remains approximately at the value ϕ_{end} . The universe evolves as a regular Friedman-Robertson-Walker (FRW) radiation-dominated universe.

TENSOR PERTURBATIONS AND RELIC GRAVITATIONAL WAVES

The phase of accelerated evolution, described in the previous section, produces a typical and unique spectrum of gravitational radiation. The basic mechanism is by now well known [23] (see [24, 25] for recent reviews). Quantum mechanical perturbations exist as tiny wrinkles on top of the classically homogeneous and isotropic background. These wrinkles are then magnified by the accelerated evolution and become classical stochastic inhomogeneities. Below I sketch the derivation of the spectrum of tensor perturbations. Many technical elements are omitted here and can be found in gory details in [1]. The classical solution (5) in conformal time η , where $dt \equiv a d\eta$, is given by

$$g_{\mu\nu} = \text{diag}(a^2(\eta), -a^2(\eta)\delta_{ij}) \quad i, j = 1, 2, 3 \quad (6)$$

where

$$a^2(\eta) \sim |\eta|^{1/2}, \quad \phi(\eta) \sim -\sqrt{3} \ln |\eta| + \phi_0 \quad (7)$$

for $\eta \rightarrow 0_-$. One expands the metric around the classical solution $g = g_{cs} + \delta g$ where g_{cs} is given in the previous equation and $\delta g_{ij} = -a^2(\eta)h_{ij}(\eta, \vec{x})$. The resulting equation of motion for each of the two independent tensor perturbation components is given in Fourier space by

$$h_k'' + 2\frac{a'}{a}h_k' + k^2 h_k = 0 \quad (8)$$

and has the general solution $h_k = A_k + B_k \ln |k\eta|$. Initial conditions corresponding to quantum fluctuations at short scales $h_k \sim 1/(a\sqrt{k}) \exp[i(\vec{k} \cdot \vec{x} - k\eta)]$, determine h_k

$$|h_k| \simeq \frac{\ln |k\eta|}{\sqrt{k}a_{HC}} \simeq \ln |k\eta|. \quad (9)$$

The amplitude of stochastic tensor perturbations in x space is characterized by $|\delta h_k| \sim k^{3/2}|h_k|$. From eq.(9) we obtain

$$|\delta h_k|^2(\eta) \sim \left(\frac{H_{max}}{M_{Pl}}\right)^2 |k\eta_{max}|^3 (\ln |k\eta|)^2. \quad (10)$$

The end of the dilaton-driven epoch is assumed [5] to take place when the curvature scale H reaches the string scale M_s . In the Einstein frame, in which M_{Pl} is constant, the string scale depends upon the dilaton as $M_s = \exp(\phi/2)M_{Pl}$. Thus we assume the dilaton-driven era to end at conformal time $|\eta| = \eta_1$ where $H_1 \simeq (\eta_1 a(\eta_1))^{-1} = M_s(\eta_1) = \exp(\phi(\eta_1)/2)M_{Pl}$. At the end of the dilaton-driven era we thus have

$$|\delta h_k(\eta_1)| \sim \frac{H_1}{M_{Pl}} (k\eta_1)^{3/2} \ln(k\eta_1) \quad (11)$$

This is the final result for the primordial spectrum of tensor perturbations. From the primordial spectrum one wishes to compute the observable spectrum today. A nice feature of gravitational waves is that gravitons are affected practically only by the evolution of the background curvature since right after the “Branch Change” era. Thus the spectrum that should be seen today should mainly reflect what happened in the very early universe processed through presumed known background evolution. While frequencies shift according to the evolution of the background scale factor throughout the evolution, amplitudes of tensor perturbations freeze while outside the horizon and evolve only when inside the horizon. If the dilaton-driven era is followed by a stringy phase characterized by an almost constant value of H , we expect scales which went out of the horizon during the dilaton-driven era to keep moving further outside and to reenter only much later, during the radiation, or possibly even matter dominated era. If we assume this to be the case for all (comoving) scales larger than η_1^{-1} , we must also assume that h_k remains frozen, for all these scales, at the value given in eq.(11) until reentry.

The result is [2] that the part of the processed spectrum which lies below a certain maximal frequency ω_{max} , the highest frequency amplified during the dilaton-driven era, is presently given, in the string frame, by

$$|\delta h_\omega| = \sqrt{H_0/M_s} z_{eq}^{-1/4} z_{out}^{1/2} \exp\left(\frac{1}{2}\phi_{end}\right) \left(\frac{\omega}{\omega_{max}}\right)^{\frac{1}{2}} \ln\left(\frac{\omega}{\omega_{max}}\right) \quad (12)$$

where $z_{out}(k) = a_{re}(k)/a_{ex}(k)$ is the red-shift while the scale k^{-1} was outside the horizon, z_{eq} is the red-shift from the matter-radiation equality epoch until today, M_s is the present value of string scale (usually estimated to be about $2-5 \cdot 10^{17} GeV$), $H_0 \sim 10^{-18} Hz$ is the present value of the Hubble parameter,

$\phi_{end} = \phi(\eta_1)$, and

$$\omega_{max} = \sqrt{H_0 M_s} z_{eq}^{-1/4} z_{out}^{-1/2}. \quad (13)$$

The fraction of energy in gravitational waves in units of the critical density is given by

$$\frac{d\Omega}{d\ln\omega} = z_{eq}^{-1} \exp(\phi_{end}) \left(\frac{\omega}{\omega_{max}}\right)^3 \ln^2\left(\frac{\omega}{\omega_{max}}\right). \quad (14)$$

Equations (12-14) were derived assuming reentry during the radiation dominated era and should be taken as good estimates and not as numerically accurate expressions. The processed spectrum of gravitational radiation is presented graphically in Figure 1,

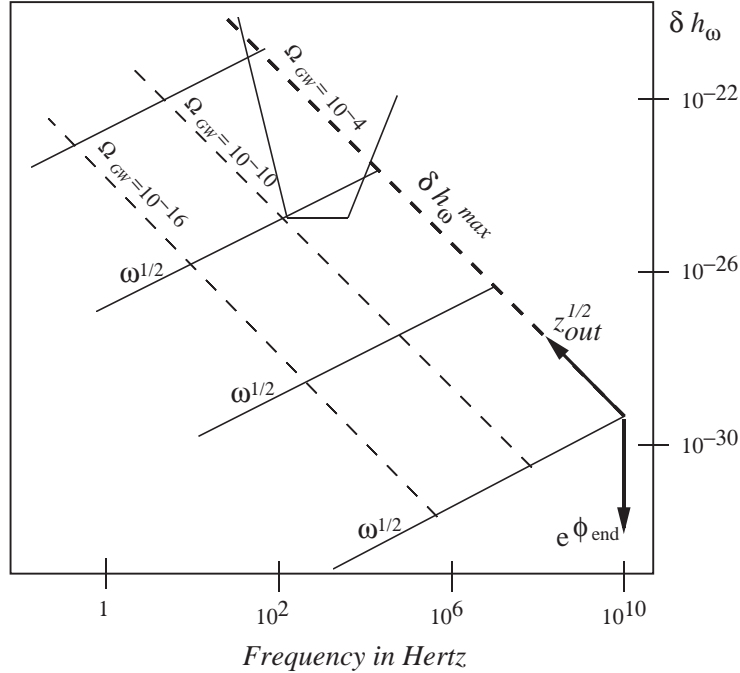


Figure 1. The characteristic spectral amplitude of gravitational waves $|\delta h_\omega|$. The solid lines show several individual spectra for different values of z_{out} and $\phi_{end} = 0$. The thick dashed line shows the maximum amplitude $|\delta h_\omega^{max}|$ as a function of z_{out} for $\phi_{end} = 0$. The dashed lines are lines of fixed ϕ_{end} and therefore lines of constant energy density. Ω_{GW} is the maximal amount of gravitational energy density at a given ϕ_{end} . Also shown in the figure is a triangular shape marking the sensitivity goals for detection of stochastic background $h_{3/yr}$, of the “Advanced LIGO”.

Two possible devices may be able to detect the predicted stochastic gravitational wave background, in the lower frequency region $1 - 10^4$ Hz, large interferometers, such as the planned LIGO[26] and VIRGO[27] and in the higher range of frequencies $10^6 - 10^9$ Hz, room-size microwave cavities. For a given set of parameters the amplitude grows as $|\delta h_\omega| \sim \omega^{1/2}$ and therefore it may seem that the best sensitivity for detection is at the high end of the spectrum $\omega = \omega_{max}$. However, the noise in a given interferometer grows as $h_n \sim \omega^{5/4}$ [28]. Therefore for a given interferometer the best sensitivity actually is in the lowest frequency range available. Microwave cavities may be operated as gravity wave detectors [29] for the high frequency range $10^6 - 10^9$ Hz. For the MHz range specific suggestions [30, 31] have been implemented [32], but not operated as gravitational radiation detector. As can be seen from Figure 1, the required sensitivity for detection at the MHz region is $h_c \sim 10^{-26}$ corresponding to $h_{3/yr}$ of the same order and therefore to a noise level of $h_n \sim 10^{-23}$ [28], assuming a bandwidth of MHz. With attainable Q factors of the order of 10^{11} , this sensitivity goal does not seem out of reach. For the GHz region the required sensitivity is $h_c \sim 10^{-28}$ corresponding to $h_n \sim 10^{-24}$.

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